# DIVISION I (GRADES K-1) Mathematics Pentathlon ${ }^{\ominus}$ Games \& Essential Resources 

## SHAPE-UP ${ }^{\text {TM }}$

The game of Shape-Up integrates fact families with geometric shape recognition, attributes, algebraic thinking, spatial visualization, and directionality. The former two abilities relate to the goal of Shape-Up. The Shape-Up gameboard is comprised of a connected network of circles which each contain a large or small version of a geometric shape. There are eight different shapes which are rotated and colored in different ways. To win the game, students are to position one of their chips on the small version and the other chip on the large version of the same shape. What enrichens this task is that in order to win students are to focus on the attributes of shape and size and disregard the rotation and color of the shape. Chip movement is based on the roll of a die. Students choose to move one or both of their chips up or down on either side of the gameboard. Strategic play is based on using fact families effectively to determine better options.

## HEX A-GONE! ${ }^{\text {TM }}$

The Hex-A-Gone gameboard is comprised of connected hexagons, trapezoids, rhombi and squares which students cover using pattern block shapes (triangles, two types of rhombi, trapezoids, squares, and hexagons). Students may use one or more blocks to fill a geometric region. For each turn students choose one, two, or three pattern blocks in an attempt to be the last player to place a block on the board (the goal). A limited number of blocks in the bank and available spaces on the gameboard motivate students to carefully observe and analyze their options. A wide array of mathematical skills are enhanced by playing this game including spatial visualization, estimation, measurement, fractions, algebraic thinking, and deductive and inductive thought.

## CAlla ${ }^{\text {tm }}$

Number sense, deductive thinking, directionality, and one-to-one correspondence are developed in this challenging game which is varied from the many ancient African and Asian counting games. In this strategy game students distribute centimeter cubes around the gameboard to plan for free turns and captures that result in acquiring the greatest number of cubes in their "Calla" (the goal).

## KINGS \& QUADRAPHAGES ${ }^{\text {¹ }}$

Horizontal, vertical, and diagonal movement, which is the basis of many mathematics/science concepts, is practiced in this simple, but challenging entrapment game. In addition, students experience a dynamic use of counting skills while exploring deductive thinking and the topology of open and closed regions. Students take turns placing a chip (referred to as a square-eating Quadraphage) on a grid-type gameboard and moving their pawn (the King) to entrap the opposing King.

## STAR TRACK ${ }^{\text {TM }}$

Number sense, inequality, addition, measurement, mapping, and decision making are experienced in this game which uses connected chain links. Students take turns selecting two chain links of various lengths from a bucket and deciding which is the better choice of the two chains. While the chain length determines how many spaces they will move on the gameboard, numeral and pentagram landings as well as bump rules help students consider that bigger is not always the better choice.

The Guide for Teaching \& Sequencing the Mathematics Pentathlon ${ }^{\circledR}$ Program for Division I (Grades K-1): This step-by-step Program Guide of Math Pentathlon lessons provides effective and detailed guidance to adults who are teaching the Math Pentathlon Program at their school. The Guide is organized into monthly lesson plans with 4 lessons per month for weekly implementation.

Adventures In Problem Solving Activity Book I (K-3): This publication connects the games with ongoing classroom and club instruction. Prerequisite skills for playing the games as well as many other problem-solving activities are described in a user-friendly format.

Investigation Exercises Book I (K-3): These problem-solving worksheets develop a more analytic focus and greater insight into the multiple strategies of each game.

## DIVISION II (GRADES 2-3)

## Mathematics Pentathlon ${ }^{\ominus}$ Games \& Essential Resources

## SUM DOMIINOES \& DICE ${ }^{\text {TM }}$

The missing addend model of subtraction, sums and differences through eighteen, algebraic thinking, spatial visualization, and deductive reasoning are concepts/skills that students experience while playing the game of Sum Dominoes \& Dice. In this strategy-chance game students search for domino faces in their hands that when attached to faces of dominoes on the gameboard equal the sum that they roll for each turn. The ability to see many different combinations for each dice-roll sum enables students to get rid of all dominoes in their hand (the goal).

## PAR $55^{\text {м }}$

This game uses attribute logic blocks to develop students' multiple classification skills, algebraic thinking, and logical and computational reasoning. The gameboard is comprised of connected pentagon bases upon which students place attribute blocks that are connected to other blocks. Whenever a "placed block" is connected to an occupied base, points are scored based on four attributes: shape, color, size, and thickness. The networking of gameboard bases, the potential for a "placed block" to be connected to multiple bases, and bump rules create a critical thinking game that challenges a broad grade and ability range.

## RAMROD ${ }^{\text {TM }}$

The game of Ramrod combines the ability to know all facts for each number family with strategic thinking. Cuisenaire rods and a gameboard that connects rectangular (sum) boxes composed of different metric lengths provide the setting for students' active investigation of addition and subtraction, measurement, algebraic thinking, estimation, spatial reasoning, and inductive and deductive thought. In this game students must plan ahead to construct "RAMROD" (addend) combinations of two rods that complete a rectangular (sum) box length in the playing area of the gameboard. Such (addend) combinations result in captures that relate to the game goal of being the first to complete their 24 cm rectangular region of the gameboard. The ability to associate each of the colored rods with their corresponding number value facilitates students' skill to mentally compute all of the facts for each number family represented on the gameboard.

## KWATRO-SINKO ${ }^{\text {м }}$

Computational and spatial reasoning as well as algebraic thinking are required in this alignment game where chips are moved along the gameboard's connected horizontal, vertical, and diagonal pathways. One player has five evennumbered chips and the other, five odd-numbered chips. The goal of the game is to create a spatial and numerical alignment which results in an answer of four or five. To play this game effectively students must know many number sentence combinations of adding two even chips and subtracting an odd numbered chip (or vise-versa) to get an answer of four or five.

## FIAR ${ }^{\text {TM }}$

This alignment game requires students to use spatial reasoning and observation skills to visualize many possible options for moving chips along the gameboard's connected horizontal, vertical, and diagonal pathways. While the rules to this game are simple, development of strategies to set up structural situations that construct winning paths and block opponent's paths are quite challenging.

The Guide for Teaching \& Sequencing the Mathematics Pentathlon ${ }^{\circledR}$ Program for Division II (Grades 2-3): This step-by-step Program Guide of Math Pentathlon lessons provides effective and detailed guidance to adults who are teaching the Math Pentathlon Program at their school. The Guide is organized into monthly lesson plans with 4 lessons per month for weekly implementation.

Adventures In Problem Solving Activity Book I (K-3): This publication connects the games with ongoing classroom and club instruction. Prerequisite skills for playing the games as well as many other problem-solving activities are described in a user-friendly format.

Investigation Exercises Book | (K-3): These problem-solving worksheets develop a more analytic focus and greater insight into the multiple strategies of each game.

# DIVISION III (GRADES 4-5) Mathematics Pentathlon ${ }^{\circledR}$ Games \& Essential Resources 

## JUGGLE ${ }^{\text {™ }}$

In this game players "juggle" polyominoes in order to be the first to complete their $9 \times 9$ grid-like gameboards. The roll of two dice determines which pieces are selected from five different types of polyominoes, including pentominoes and tetrominoes. In the process of playing the game students develop a deeper understanding of mathematical relationships involving area, perimeter, and transformational geometry (flips/reflections, turns/ rotations, and slides/translations). Spatial and logical reasoning skills are enhanced as students try to balance probability and chance, as well as numerical and structural relationships.

## CONTIG 60 ${ }^{\text {™ }}$

The four basic operations (addition, subtraction, multiplication, and division), probability, and algebraic thinking are integrated in this strategy-chance game. Numerical values are arranged in a particular pattern on the gameboard. For each play students form number sentences based on the roll of 3 regular dice and place chips on corresponding numbered spaces. Arranging 5 chips along contiguous horizontal, vertical, or diagonal lines relates to one goal of the game. A second goal pertains to scoring and connected chip placement. By playing this game students practice basic skills in a highly motivational format that also enhances their problem-solving and computational reasoning abilities.

## STARS \& BARS ${ }^{\text {TM }}$

This multiple classification-logic game uses geometric attribute cards and a grid-type board to help students develop understanding of multi-variable relationships and algebraic principles. For each play students try to maximize their scores by placing cards onto the gameboard and comparing them to adjacent cards that are one, two, three, or four-ways different. The horizontal, vertical, and diagonal placement of the cards onto the gameboard contributes to the development of logical, computational, and spatial thought.

## FAB-A-DIFFY ${ }^{\mathrm{TM}}$

Fraction bars that pictorially represent various fraction values are used in this computational reasoning game. In playing this game students attempt to make arithmetic combinations on the specially designed gameboard. Conceptual understanding of equivalence and addition and subtraction of fractions as well as algebraic principles are developed while students connect symbolic procedures with visual models.

## QUEEN'S $8^{\circ}$ GUARDS ${ }^{\text {TM }}$

This spatial reasoning game combines the simplicity of rules found in Checkers with the complexity of strategy experienced in Chess. A gameboard of tessellated hexagons that form concentric bands and pawns (queens) and chips (guards) are used to simulate the agonistic pattern of bees in a beehive. By learning how to construct a variety of triangular and other structural arrangements on the gameboard, students develop spatial thinking as well as many other logical reasoning skills, including deductive and inductive thought.

## The Guide for Teaching \& Sequencing the Mathematics Pentathlon ${ }^{\circledR}$ Program for

Division III (Grades 4-5): This step-by-step Program Guide of Math Pentathlon lessons provides effective and detailed guidance to adults who are teaching the Math Pentathlon Program at their school. The Guide is organized into monthly lesson plans with 4 lessons per month for weekly implementation.
Adventures in Problem Solving Activity Book II (Grades 4-7): This publication connects the games with ongoing classroom and club instruction. Prerequisite skills for playing the games as well as many other problem-solving activities are described in a user-friendly format.

Investigation Exercises Book II (Grades 4-7): These problem-solving worksheets develop a more analytic focus and greater insight into the multiple strategies of each game.

# DIVISION IV (GRADES 6-7) <br> Mathematics Pentathlon ${ }^{\circledR}$ Games \& Essential Resources 

## PRIME GOLD ${ }^{\text {TN }}$

The four basic operations, exponentiation, factorials, algebraic thinking, and prime and composite numbers are integrated in this strategy-chance alignment game. The gameboard represents a grid with numerical values that sequentially spiral from the board's center to the outer band which results in prime numbers that occur along diagonal lines. For each play students form number sentences based on the roll of 3 polyhedra dice and place chips on corresponding numbered spaces of the gameboard. The structural/numerical arrangement of the gameboard relates to one of the games goals: constructing 4 diagonal veins of prime gold. Blocking rules dealing with prime and composite numbers, multiple chip placement based on Goldbach's conjecture, and a second game goal make this a most challenging computational reasoning game.

## REMAINDER ISLANDS ${ }^{\text {TM }}$

Polyhedra dice, pawns, chips and a hexagonal-triangular shaped gameboard are used to play this strategy-chance division game. The goal is to place the greatest number of chips on the board's six hexagonal-shaped islands. A series of rules related to zero remainders and networking pawns along strategic pathways make this both a conceptual and problem-solving game.

## PENT 'EM IIY ${ }^{\text {TM }}$

This transformational geometry game develops spatial visualization as well as both deductive and inductive reasoning. Each player uses 5 pentominoes (chosen from a set of 12 pieces) and two blocking chips on a 13 by 9 grid to entrap all 5 of the opponent's pentominoes. In the placement phase players must strategically select and place five pentomino pieces onto the gameboard. In the movement phase players attempt to entrap the opposing player's five pentominoes by rotating, flipping, or sliding their own pieces.

## FRAC FACT ${ }^{\text {mw }}$

Equivalence, the four basic operations as well as algebraic principles are investigated in this conceptual and strategic fraction game. A set of 64 fraction bars that pictorially represent various fraction values are used to help students connect symbolic procedures with visual models. On each turn students try to find two bars that when combined through the four basic operations equal one of the answer bars on the specially designed gameboard. Students must use strategic thinking skills when examining all possible combinations of bars and operations as well as setting up moves for future options.

## FRACTION PINBALL ${ }^{\text {TM }}$

The gameboard and game pieces in this fraction-decimal conversion game simulate the ricocheting action of pinballs in a pinball machine. In playing this game students must strategically position and move pawns and blocking chips to land on fraction values of the gameboard which are converted to decimal form and are added to or subtracted from an accumulated score. The goal of the game is to be the first player to reach an accumulated score of between 4.95 and 5.05 . Since the gameboard is composed of a network of interconnected fraction circles, students must use both spatial and computational reasoning when making decisions.

## The Guide for Teaching \& Sequencing the Mathematics Pentathlon ${ }^{\circledR}$ Program for Division IV

(Grades 6-7): This step-by-step Program Guide of Math Pentathlon lessons provides effective and detailed guidance to adults who are teaching the Math Pentathlon Program at their school. The Guide is organized into monthly lesson plans with 4 lessons per month for weekly implementation.

Adventures in Problem Solving Activity Book II (Grades 4-7): This publication connects the games with ongoing classroom and club instruction. Prerequisite skills for playing the games as well as many other problem-solving activities are described in a user-friendly format.

Investigation Exercises Book II (Grades 4-7): These problem-solving worksheets develop a more analytic focus and greater insight into the multiple strategies of each game.

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|  | 7 | SumDoninoes \＆Dicem ${ }^{\text {m }}$ | t | ＊ | $\star$ | $\star$ | JT |  | TT | ＊ | t | ＊ | ＊ |  |  |  |  |  |  |  |  |  |  | $\star$ | ＊ |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ |  | ＊ |  | $\star$ | $\star$ |  | $\star$ |  |  |  |  |  |
|  | 8 | Ramrod ${ }^{\text {m }}$ | t | IT | ＊ | $\star$ | JT |  | $\star$ | $\star$ |  | ＋ | $\star$ |  |  |  |  |  | $\star$ |  |  |  |  |  | ＊ |  | $\star$ | ＊ | ＊ | $\star$ | ＊ | $\pi$ | $\star$ |  | $\star$ | ＊ | $\star$ | $\star$ | T |  |  | ＊ | IT |
|  | 9 | PAR $55^{\text {m }}$ | ＊ | ＊ | $\star$ | $\star$ |  |  | $\star$ | ＊ |  | ＊ | $\star$ |  |  |  |  |  |  |  |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | ＊ | ＊ | $\star$ |  | ＊ | $\star$ | $\star$ |  | $\star$ | $\star$ |  |  | $\star$ |  |  |
|  | 10 | Kwatro－Sinkom ${ }^{\text {m }}$ | t |  | － | ＊ |  |  | JT | J |  | ＋ | ＊ |  |  |  |  |  |  | $\star$ |  |  |  |  | ＊ |  | $\star$ | ＊ | ＊ | $\star$ | $\star$ |  | ＊ |  | ＊ | $\star$ | ＊ | ＊ | IT |  |  |  | IT |
| $\begin{aligned} & \bar{B} \\ & \text { Z } \\ & \frac{1}{n} \\ & \bar{i} \end{aligned}$ | 11 | contig $60{ }^{\text {m }}$ | $\star$ |  | TT | t |  |  | $\star$ | ＊ |  | ＊ | $\star$ | ＋ | t | ＋ |  |  |  |  |  |  |  | $\star$ | $\star$ |  | $\star$ | $\star$ | ＊ | $\star$ | $\star$ | ＊ | ＊ |  | $\star$ |  |  | ＊ |  |  |  |  |  |
|  | 12 | Jugglem | $\star$ |  | TT | IT |  |  | TT | T |  | $\star$ | $\star$ | ＊ | ＊ | ＋ |  |  | $\star$ |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ |  | $\star$ | $\star$ |  | $\star$ |  | ＊ | $\star$ | $\star$ |  | $\star$ | $\star$ |
|  | 13 | Stars \＆Bars ${ }^{\text {m }}$ | $\star$ |  |  | $\star$ |  |  | $\star$ | ＊ |  | ＊ | $\star$ | ＊ |  |  |  |  |  |  |  |  |  |  | $\star$ | ＊ | $\star$ | ＊ | ＊ | K | ＊ |  | t | $\star$ | $\star$ |  |  | ＊ |  |  |  |  |  |
|  | 14 | Queens \＆Guards ${ }^{\text {m }}$ | $\star$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\star$ |  | $\star$ | $\star$ | ＊ | ＊ | $\star$ |  | ＊ | $\star$ | $\star$ |  |  | $\star$ | $\star$ |  |  |  |  |
|  | 15 | Fab－ADDiffy ${ }^{\text {m }}$ | $\star$ |  | $\star$ | t |  |  | $\star$ | ＊ |  | $\star$ | $\star$ |  |  |  | $\star$ |  |  |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ | ＊ | ＊ | ＊ |  |  |  | $\star$ |  |  | ＊ |  |  |  | $\star$ |  |
| $\begin{aligned} & \geq \\ & 2 \\ & 0 \\ & \vdots \\ & \vdots \end{aligned}$ | 16 | Remainder slands $^{\text {m }}$ m | $\star$ | ＊ |  |  |  |  |  | $\star$ | ＋ | ＊ | $\star$ | ＋ | t | ＋ |  |  |  |  |  |  |  | ＊ | $\star$ |  | $\star$ | $\star$ | $\star$ | K | ＊ |  | $\star$ | ＊ | ＋ | $\star$ | $\star$ | ＊ |  |  |  |  |  |
|  | 17 | Prime Gold ${ }^{\text {m }}$ | $\star$ |  | $\pi$ | $\star$ |  |  | $\star$ | $\star$ |  | ＋ | $\star$ | $\star$ | ＊ | ＊ |  |  |  | $\star$ |  |  | $\star$ | $\star$ | $\star$ |  | $\star$ | $\star$ | $\star$ | ＊ | $\star$ | $\star$ | $\star$ |  | $\star$ |  |  | $\star$ |  |  |  |  |  |
|  | 18 | Pent ${ }^{\text {Emin }}{ }^{\text {m }}$ | $\star$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\star$ | ＊ | $\star$ | $\star$ | $\star$ | ， | ＊ |  | ＊ | ＊ | ＊ | $\star$ |  | t | $\star$ | ＊ |  | ＊ | $\star$ |
|  | 19 | Frac Fad ${ }^{\text {m }}$ | ＊ |  | $\star$ | $\star$ |  |  | $\star$ | ＊ | ＊ | $\star$ | $\star$ | $\star$ | t | ＋ | $\star$ |  |  |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ | ＊ | $\star$ |  | t | $\star$ | $\star$ |  |  | $\star$ |  |  |  | $\star$ |  |
|  | 20 | Fraction Pinball ${ }^{\text {m }}$ | $\star$ |  |  | t |  |  | $\star$ | ＋ |  | $\star$ | $\star$ |  |  |  | $\star$ | $\star$ |  |  |  |  |  |  | $\star$ |  | ＋ | ＋ | ＊ | ＋ | $\star$ |  | $\star$ | $\pm$ | ＋ | ＋ | ＊ | t |  |  |  |  |  |


|  |  |  |  | COMPUTATIONAL RGASONING |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | LOGICAUSCIGNTIFIC RGASONING |  |  |  |  |  |  |  | SPATIALGGOMETRIC RGASONING |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 喜 <br> 言 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | （ex |  |
|  | 1 | Kings \＆Quadraphages ${ }^{\text {m }}$ | $\star$ | ＊ | $\star$ | $\star$ |  | ＊ | $\star$ | $\star$ |  |  |  |  |  |  |  |  |  |  |  |  |  | $\star$ | T | $\star$ | $\star$ | $\star$ |  |  | $\star$ | ＊ | ＊ |  | $\star$ | ＊ |  | $\star$ |  |
|  | 2 | Calla ${ }^{\text {m }}$ | $\star$ | 戍 | $\star$ | $\star$ | Tr | ＊ | $\star$ | $\star$ | $\star$ |  |  |  |  |  |  |  |  |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ | ＋ |  | $\star$ | ＊ |  |  |  |  |  |  |  |
|  | 3 | HexAGone！m | $\star$ | $\star$ | $\star$ | $\star$ |  | $\star$ | $\star$ | $\star$ | $\star$ | ＊ |  |  |  | $\star$ |  | ， |  |  |  |  |  | $\star$ | ＊ | $\star$ | $\star$ | $\star$ | ＋ | $\star$ | $\star$ | $\star \pi$ |  |  | $\star$ | $\star$ | ＊ | $\star$ | ＊ |
|  | 4 | Shape－Lp ${ }^{\text {m }}$ | $\star$ | 大 | $\star$ | $\star$ |  | ＊ | $\star$ | $\star$ |  | ＋ |  |  |  |  |  |  |  |  |  |  |  | $\star$ | ＊ | $\star$ | 大 | $\star$ | ＊ |  | $\star$ | $\star$ |  |  |  |  | $\star \star$ |  | $\star$ |
|  | 5 | Star Track ${ }^{\text {m }}$ | $\star$ | t | ＊ |  | ＊ | ＊ | $\pm$ | ＋ | ＊ |  |  |  |  |  |  |  |  |  |  |  |  | $\star$ | $\star$ | 太 | ＊ | ＊ |  |  |  | t |  |  |  |  |  |  |  |
| $\begin{aligned} & \bar{Z} \\ & \underline{0} \\ & \underline{n} \\ & \overline{0} \end{aligned}$ | 6 | FARTM | $\star$ |  | T |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ |  |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ | $\pi$ | $\pi$ |  | IT |
|  | 7 | SumDominoes \＆Dice ${ }^{\text {mm }}$ | $\star$ | $\star$ | $\star$ | $\star$ | T |  | IT | $\star$ | $\star$ | t |  |  |  |  |  |  |  |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ | ＊ |  | $\star$ | $\star$ | $\star$ |  | $\star$ |  |  |  |  |
|  | 8 | Ramrodm ${ }^{\text {m }}$ | $\star$ | j | $\star$ | $\star$ | T |  | $\star$ | $\star$ | $\star$ | t |  |  |  |  |  | $\star$ |  |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ | ＊ | $\pi$ | $\star$ | ＊ | $\star$ | $\star$ |  | Tr |  | $\star$ | IT |
|  | 9 | PAR $55^{\text {m }}$ | $\star$ | 大 | $\star$ | $\star$ |  |  | $\star$ | $\star$ | $\star$ | － |  |  |  |  |  |  |  |  |  |  |  | $\star$ | ＊ | $\star$ | $\star$ | $\star$ | $\star$ |  | ＊ | $\star$ |  | $\star$ | $\star$ |  | $\star$ |  |  |
|  | 10 | Kwatro－Sinkom ${ }^{\text {ma }}$ | ＊ |  |  | ＊ |  |  |  | $\pi$ |  | － |  |  |  |  |  |  | t |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | ＊ | ＋ |  | ＊ | $\star$ | ＊ | $\star$ |  | $\pi$ |  |  | $\pi$ |
|  | 11 | Contig 60 mm | ＊ |  |  | ＊ |  |  | $\star$ | ＊ | $\star$ | ＋ |  | ＊ | ＊ |  |  |  |  |  |  |  |  | $\star$ | ＊ | $\star$ | $\star$ | $\star$ | ＊ | $\star$ | ＊ | $\star$ |  |  | $\star$ |  |  |  |  |
|  | 12 | Juggle ${ }^{\text {m }}$ | $\star$ |  |  | T |  |  |  | J |  | － |  | $\star$ | $\star$ |  |  | $\star$ |  |  |  |  |  | $\star$ | ＊ | $\star$ | $\star$ | $\star$ |  |  | $\star$ | $\star$ | $\star$ |  | $\star$ | $\star$ |  | $\star$ | $\star$ |
|  | 13 | Stars \＆Ears ${ }^{\text {mm }}$ | $\star$ |  |  | $\star$ |  |  | $\star$ | $\star$ |  | ＊ |  | $\star$ |  |  |  |  |  |  |  |  |  | $\star$ | ＊ | $\star$ | $\star$ | $\star$ |  |  | $\star$ | ＊ |  |  | $\star$ |  |  |  |  |
|  | 14 | Queens \＆Guards ${ }^{\text {m }}$ | $\star$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ |  |  | $\star$ | $\star$＊ |  |  | $\star$ | $\star$ |  |  |  |
|  | 15 | Fab－AD－iffy ${ }^{\text {m }}$ | $\star$ |  |  | $\star$ |  |  | $\star$ | $\star$ |  | ＊ |  |  |  | ＋ |  |  |  |  |  |  |  | $\star$ | ＊ | $\star$ | $\star$ | $\star$ |  |  | ＊ | $\star$ t |  |  | $\star$ |  |  | $\star$ |  |
| $\begin{aligned} & \geq \\ & 2 \\ & 0 \\ & \frac{0}{\lambda} \\ & i \end{aligned}$ | 16 | Remainder Islands ${ }^{\text {m }}$ | ＊ | ＊ |  |  |  |  |  | $\star$ | $\star$ | ＋ |  | ＊ | $\star$ |  |  |  |  |  |  |  |  | $\star$ | $\star$ | $\star$ | $\star$ | $\star$ |  |  | $\star$ | t | ＊ | $\star$ | $\star$ |  |  |  |  |
|  | 17 | Prime Goldm | $\star$ |  |  | $\star$ |  |  | $\star$ | $\star$ | $\star$ | － |  | ＊ | $\star$ |  |  |  | ＊ |  |  | $\star$ |  | $\star$ | ＊ | $\star$ | $\star$ | $\star$ | 大 | $\star$ | $\star$ | $\star$ |  |  | $\star$ |  |  |  |  |
|  | 18 | Pent ${ }^{\text {Em }}$／ $\mathrm{m}^{\text {m }}$ | $\star$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\star$ | ＊ | $\star$ | $\star$ | $\star$ | 夫 |  | $\star$ | $\star$ | $\star$ |  | $\star$ | $\star$ |  | ＊ | $\star$ |
|  | 19 | Fracrad ${ }^{\text {m }}$ | $\star$ |  | $\star$ | $\star$ |  |  | $\star$ | $\star$ | $\star$ | ＊ |  | $\star$ | $\star$ | $\star$ |  |  |  |  |  |  | ＊ | $\star$ | － | $\star$ | $\star$ | $\star$ | ＋ |  | $\star$ | $\star$ |  |  | $\star$ |  |  | $\star$ |  |
|  | 20 | Fraction Pinball ${ }^{\text {m }}$ | $\star$ |  |  | $\star$ |  |  |  |  |  |  |  |  |  | ત | ＋ |  |  |  |  |  |  | $\star$ | 大 | ＊ | ＊ | 大 | K |  | $\star$ | $\star$ | $\star$ | ＊ | $\star$ |  |  |  |  |

