Mathematics Defined

Many times teachers and parents will define mathematics as a study of numbers and arithmetic operations. Did your definition look at mathematics from this perspective? Other definitions focus on mathematics as a tool or collection of skills that can be applied to questions of "how many" or "how much." These definitions quite often concentrate on the application of calculation skills to everyday problems involving quantification. Some adults comment that they must not be good at math because they can not balance their checkbooks! Another view of mathematics is that it is a science which involves logical reasoning, drawing conclusions from assumed premises, systematized knowledge, and/or strategic reasoning based on accepted rules, laws, or probabilities (decision analysis). Mathematics might also be defined as an art which studies patterns for predictive purposes. Another common view of mathematics is that it is a specialized language which deals with form, size, and quantity. Did your definition incorporate any of these ideas?

In examining mathematics from a historical perspective, one can see that much of its development was directed to describing patterns of relationship that were of interest to various individuals. In studying these patterns, a need for a specialized set of symbols (idiograms) developed. At the same time as these codes were being developed, a set of grammatical rules evolved for the appropriate use of these symbols. The early history of mathematics is repleat with examples of human interest in patterns (concepts) which directly related to common experiences or phenomena.

As the field of mathematics matured, the symbols and grammatical rules began to form a systematic structure that became a subject of study in itself. From these events, two types of mathematicians emerged: those known as *applied mathematicians* who were interested in how the symbols and codes could be related to human experiences or phenomena (isomorphisms), and those refered to as *pure mathematicians* who devised systems of symbols to study the ramifications of selecting a particular set of rules (axioms). Although pure mathematicians generally show no interest in determining if their abstract studies have any relevance to human experiences or phenomena, applied mathematicians often study the work of pure mathematicians to see if their abstract systems might be linked to or explain the relationships underlying certain observed patterns. This synergistic relationship between pure and applied mathematicians has enabled us to mature in our understandings about various fields of human endeavor and has added immeasurably to our technological sophistication.

The major concern addressed by the Pentathlon Institute is what should constitute the mathematics curriculum and how should it be presented in educating our students. We believe that the major thrust of a mathematics curriculum from kindergarten through college should stress **mathematical literacy** so that each student can develop an understanding of how mathematics is relevant to the fields of endeavor they may eventually choose for themselves. This means that we must view our students as either potential applied mathematicians who may use mathematics to enhance their own selected fields of endeavor or literate consumers of products from applied mathematics, capable of understanding the mathematics used within their field.

 Mary Gilfeather & John del Regato

We do not believe that it serves society well to provide a mathematics curriculum from kindergarten through college designed to develop pure mathematicians. Rather, we believe that learning experiences should be designed for students that provide greater potential for developing attitudes and thought processes consistent with being a user of or a consumer of applied mathematics. The very small percentage of students showing an interest in pure mathematics should be provided with additional mathematics courses that are taught by qualified pure mathematicians. Furthermore, the much larger group of potential applied mathematicians should be given a much richer variety of mathematical experiences then they are currently receiving.

From the applied mathematicians' perspective, the symbols and codes of mathematics are not the concepts themselves but are an abstract representation of the concepts being studied. In order to understand and remember the underlying concepts which the symbols represent, physical and pictorial models can be used. Such models help students interpret patterns and relationships, connect these ideas to symbolic codes, and provide a means to remember these ideas through visual memory.

The definition of mathematics used as a basis for **MEBA**™ is one which looks at both the processes and products involved in thinking mathematically. It is our belief that:

MATHEMATICS IS AN AREA OF INVESTIGATION
WHICH LOGICALLY ANALYZES
ORDERING,
OPERATIONAL,
AND STRUCTURAL RELATIONSHIPS.

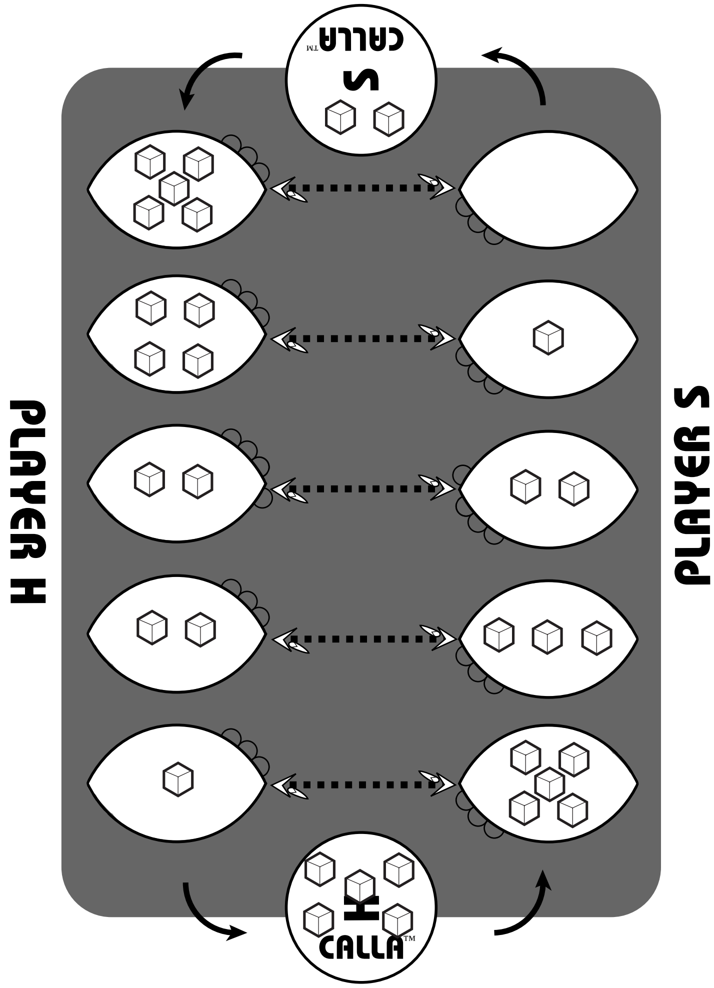
It is true that a specialized set of symbols and grammatical rules were designed to communicate mathematical ideas and relationships. However, the symbols are not the concepts! They are used to convey the concepts.

According to our definition then, there must be something to investigate. Neither meaningless memorization nor symbol manipulation is seen as legitimate mathematics. Instead mathematical thinking is directly related to three types of relationships:

ORDERING RELATIONSHIPS
OPERATIONAL RELATIONSHIPS
STRUCTURAL RELATIONSHIPS

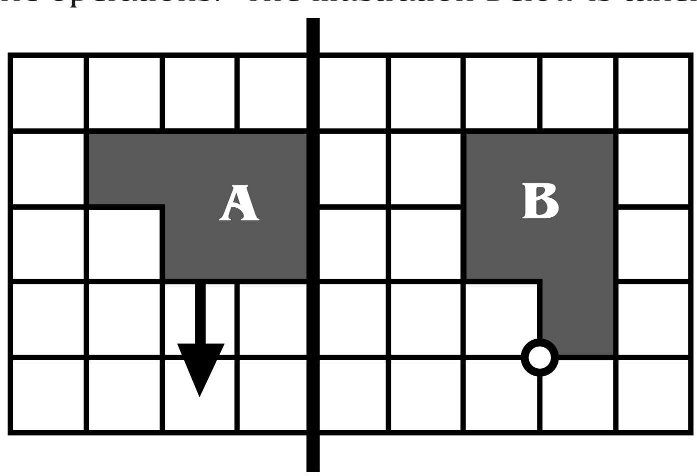
ORDERING RELATIONSHIPS concern sequencing ideas of a positional, categorical, or deductive nature. Tasks that are positional in nature include arranging numbers from smallest to largest or placing pictures of children in order from the greatest height to the least height. Repeating patterns also involve positional issues. For example, a student could be given a cube color sequence such as the one illustrated and be asked to complete or add on to the sequence with additional black and white cubes. Sorting a set of objects by a particular attribute e.g. color, or shape is an example of the categorical nature of ordering relationships.

The deductive nature of ordering relationships occurs when many components must be arranged in a particular order for successful completion of a problem. An example of a problem solving task that uses deductive thought comes from ***Investigation Exercises Book I*** (Grades K - 3). See illustration at right. (This publication provides many nonroutine problem-solving tasks that relate to the **Mathematics Pentathlon**®, a series of interactive problem-solving games. See pages 20 and 22 for a detailed description.)



This problem requires students to analyze a mid-game situation and determine which player will win if they both play a perfect game (make no mistakes). To solve this, students must use deductive thought to sequence a series of free turns and captures in the game of Calla™.

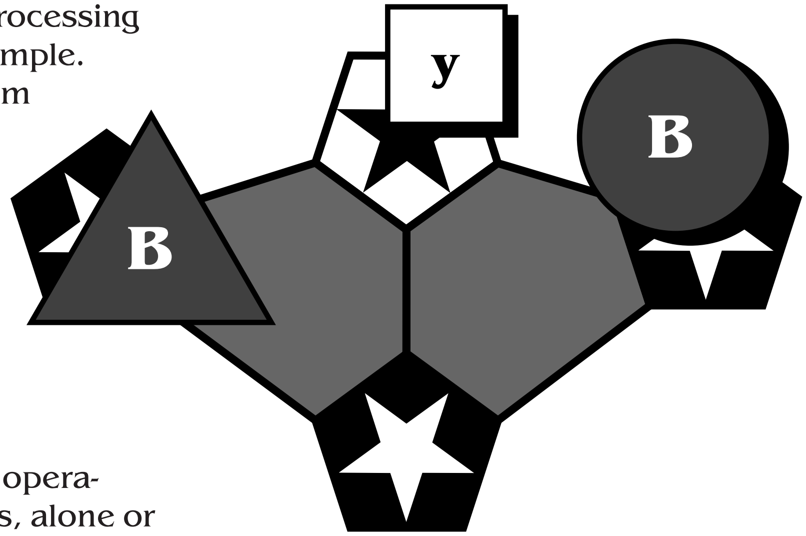
OPERATIONAL RELATIONSHIPS involve the interconnectedness between various ways in which the elements of a problem can be acted upon. For example, is there some relationship between addition and subtraction or addition and multiplication? How would you describe that interconnectedness? What about subtraction and division? In transformational geometry, how would you describe the relationship between the operations of flip (reflection), slide (translation), and rotation? Such questions explore the relationship between various arithmetic or geometric operations. The illustration below is taken from ***Investigation Exercises Book II (Grades 4 - 8)***. To develop their spatial reasoning skills, students study the effects of three transformational geometry operations. Position A represents the starting position and Position B indicates the ending position. The pentomino in Position A resulted in Position B from a combination of three transformational operations: a flip (reflection), a slide (translation), and a rotation. Is the order in which these operations are performed important?



STRUCTURAL RELATIONSHIPS concern multi-variable or multifaceted ideas. How

 Mary Gilfeather & John del Regato

one element within a structure simultaneously relates to other elements within that structure is the essence of this type of thinking. Investigating two-dimensional or three-dimentional geometric patterns can be used as an example of learning structural relationships. Multi-classification tasks which require the simultaneous processing of several variables is another example. The research question taken from ***Investigation Exercises Book I (Grades K - 3)*** illustrates how students are studying structural relationships when engaged in the **Mathematics Pentathlon**® game of PAR 55™.



In **logically analyzing** ordering, operational, and structural relationships, alone or in a group, an individual is thinking mathematically. To logically analyze means that all given relevant information in a problem must be used. In other words, relevant information may not be ignored just because it doesn't fit certain generalizations. The generalizations or statements of relationship must accommodate all known, relevant aspects of the problem.